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L.G. Taff

Relativity and Radars

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FOR THE COMMANDER

Thomas J. Alpert

Thomas J. Alpert, Major, USAF
Chief, ESD Lincoln Laboratory Project Office

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

RELATIVITY AND RADARS

L.G. TAFF
Group 94

TECHNICAL REPORT 660

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LEXINGTON MASSACHUSETTS

Abstract

This Report discusses the special relativistic and general relativistic effects on electromagnetic propogation for artificial satellite applications. In particular the rigorous special relativistic Doppler shift between the received and transmitted frequencies is derived. The two radars need not be colocated and are assumed not to be in the same inertial reference frame. Numerical estimates of various approximations, second order series approximations, and lowest order post-Newtonian general relativistic terms are all performed. This work was spurred by the observational determination of the $(v/c)^2$ term in Millstone Hill Radar observations of LAGEOS.

CONTENTS

I. BASIC FORMULAS

The central formula from the theory of special relativity pertinent to this discussion is the Doppler shift equation. This result may be derived in a variety of ways and may be looked upon as an expression of the coordinate system invariance of the phase of an electromagnetic wavetrain. As the phase is dependent upon the scalar product between two 4-vectors, we are used to seeing the Doppler shift results displayed as two equations--one from the temporal part of the 4-vector scalar product and one from the spatial part. Actually, because of the nature of the Lorentz transformation for a 4-vector, the Doppler shift formulas can be broken into three parts.

Consider two inertial reference frames in relative motion. Let their relative velocity be given by the 3-vector $\underline{v} = c\underline{\beta}$ where c is the speed of light. Define γ as

$$\gamma = (1-\beta^2)^{-1/2}$$

Then the Lorentz transformation of an arbitrary 4-vector $A = (A_0, A_1, A_2, A_3) = (A_0, \underline{A})$ is given by

$$a_0 = \gamma(A_0 - \underline{\beta}\cdot\underline{A})$$

$$a_\parallel = \gamma(A_\parallel - \beta A_0)$$

$$\underline{a}_\perp = \underline{A}_\perp$$

where one observer measures $A = (A_0, \underline{A})$ and the other inertial observer perceives $a = (a_0, \underline{a})$. The notation $\parallel$ and $\perp$ refer to directions parallel to and perpendicular to $\underline{v}$.

For a plane wave of frequency F and wave vector $\underline{K}$ the result is

$$f = \gamma(F - \underline{\beta}\cdot\underline{K})$$

$$k_{\parallel} = \gamma(K_{\parallel} - \beta F)$$

$$\underline{k}_{\perp} = \underline{K}_{\perp}$$

Because of the special nature of light waves in special relativity these may be more simply written as $(F=Kc)$

$$f = \gamma F(1 - \beta\cos\Theta)$$

$$\tan\theta = \frac{\sin\Theta}{\gamma(\cos\Theta-\beta)} \tag{1}$$

where Θ (θ) is the angle between $\underline{K}$ ($\underline{k}$) and $\underline{v}$. The inverse to Eqs. (1) may be obtained by interchanging lower case and upper case letters and changing the sign of β.

Another way to describe the meaning of the angle Θ (θ) is that if the inertial observer (the inertial observer moving with velocity $\underline{v}$) perceives the direction of propogation of the ray to be along the unit vector $\underline{N}$ ($\underline{n}$), then $\beta\cos\Theta = \underline{\beta}\cdot\underline{N}$ ($\beta\cos\theta = \underline{\beta}\cdot\underline{n}$). In this notation

$$f = \gamma F(1 - \underline{\beta}\cdot\underline{N})$$

$$= (F/\gamma)/(1 + \underline{\beta}\cdot\underline{n}) \tag{2}$$

The second of Eqs. (1) is an aberration effect that is present even if the two observers are instantaneously colocated.

II. THE SCENARIO

The transmitting radar emits a pulse of radiation out into space. This transmitted pulse (T subscript below) is reflected by an artificial satellite sometime later. Even later the reflected pulse is intercepted by the receiving radar. See Fig. 1. The receiving radar may be colocated with the transmitting radar (possibly identical with it) or not. The transmitting radar knows the frequency of the transmitted pulse (f_T) as measured in the inertial reference frame which is instantaneously comoving with it at the time of transmission. Continuing in this anthropomorphic vein, the transmitting radar also knows the direction of propogation of the outgoing pulse as perceived in this same inertial frame (unit vector = $\underline{n}_T$).

The receiving radar has available to it similar knowledge concerning the received pulse (R subscript below). In particular this radar observes an incoming pulse of frequency f_R coming from a direction $\underline{n}_R$. Both radars are also aware of their velocities (at the time of transmission and reception respectively) relative to an inertial observer located at the center of the Earth. Indeed, the geocentric inertial observer perceived the transmitting radar, instantaneously moving with a velocity $\underline{v}_T = c\underline{\beta}_T$, to transmit a pulse of frequency F_T in a direction $\underline{N}_T$. Sometime later an artificial satellite with velocity $\underline{v}_S = c\underline{\beta}_S$ (relative to the geocentric inertial frame) reflected this pulse. Later still the geocentric inertial observer perceived the intercepting radar, instantaneously moving with a velocity $\underline{v}_R = c\underline{\beta}_R$, to receive a pulse of frequency F_R from the direction $\underline{N}_R$.

The desideratum in this problem is to express the frequency component of the Doppler shift, $\Delta f = f_R - f_T$, in terms of known (the velocities) or observed (the directions and the frequencies) quantities.

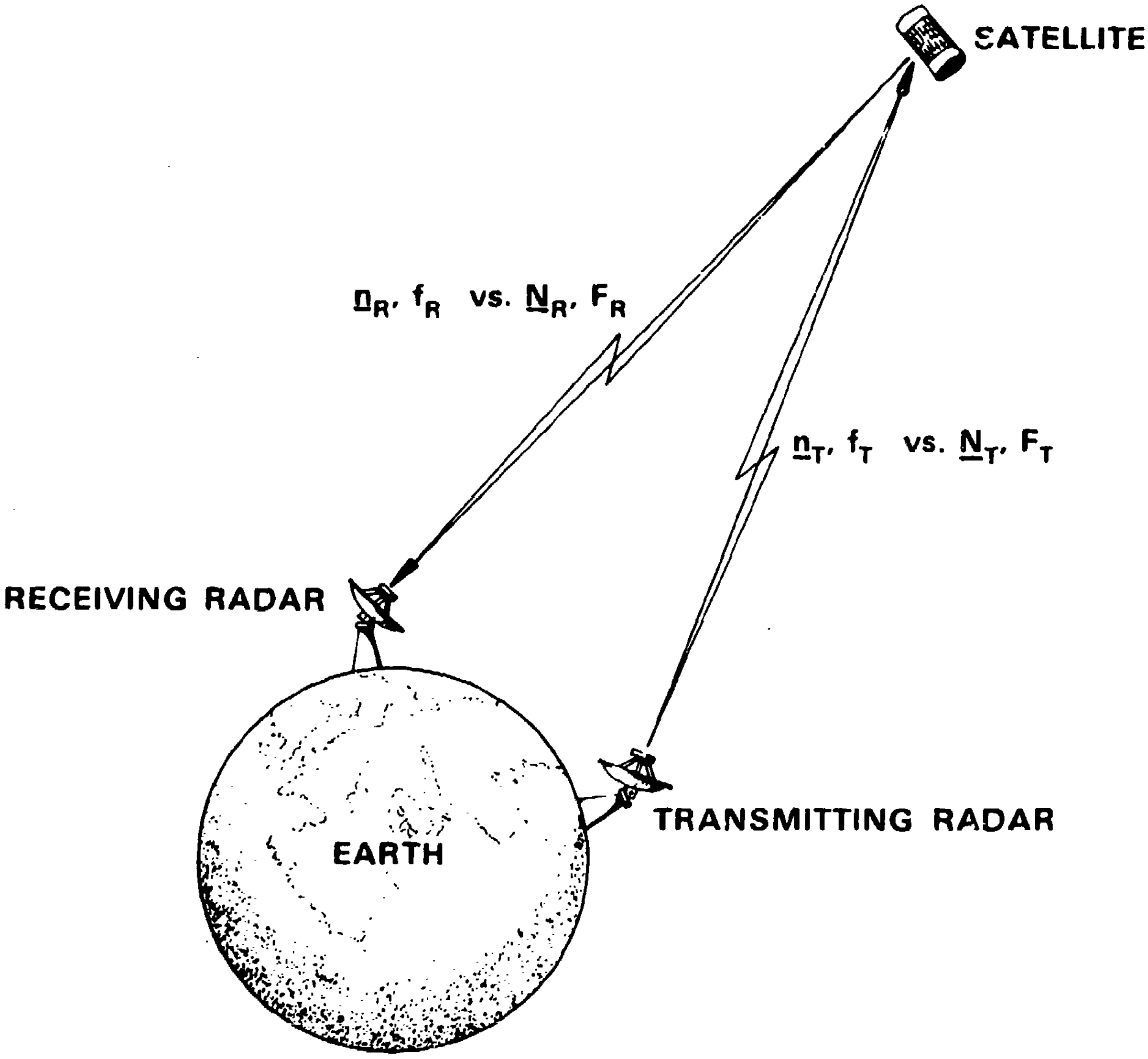

Fig. 1. Schematic representation of the transmitted (T) and received (R) radar signals. The displacement may be due to the Earth's rotation or the non-colocated radars.

III. RIGOROUS SPECIAL RELATIVISTIC RESULT

Define γ_R, γ_T by

$$\gamma_R = (1 - \beta_R^2)^{-1/2}$$

$$\gamma_T = (1 - \beta_T^2)^{-1/2}$$

Then from Eq. (2) the relationship between the geocentric inertial observer's perception of the frequency of the transmitted pulse (F_T) and the frequency detected by an observer instantaneously comoving with the transmitting radar (f_T) is

$$f_T = F_T \gamma_T (1 - \underline{\beta}_T \cdot \underline{N}_T)$$

$$= (F_T/\gamma_T)/(1 + \underline{\beta}_T \cdot \underline{n}_T) \qquad (3)$$

The next step in the scenario is the reflection of this outgoing pulse by an artificial satellite. In the geocentric inertial reference frame the artificial satellite has a velocity $\underline{v}_S = c\underline{\beta}_S$ and reflected the transmitted pulse in the direction $\underline{N}_R$. An observer instantaneously comoving with the artificial satellite perceived the transmitted pulse approaching with frequency f_S,

$$f_S = F_T \gamma_S (1 - \underline{\beta}_S \cdot \underline{N}_T)$$

where $\gamma_S = (1 - \beta_S^2)^{-1/2}$. The reflected pulse receded from the satellite's surface, relative to the geocentric inertial observer, at a frequency F_R.

This frequency is related to f_S via

$$f_S = F_R \gamma_S (1 - \underline{\beta}_S \cdot \underline{N}_R)$$

So, utilizing the last two formulas, I can write

$$F_R = (f_S/\gamma_S)/(1 - \underline{\beta}_S \cdot \underline{N}_R)$$

$$= F_T (1 - \underline{\beta}_S \cdot \underline{N}_T)/(1 - \underline{\beta}_S \cdot \underline{N}_R) \qquad (4)$$

It is important to note that the relationship between the received and transmitted pulse frequencies (in the geocentric inertial frame) contains two extra special relativistic components. One is equal to γ_S, the other to $1/\gamma_S$. Hence their cumulative effect is nil and the final relationship has the appearance of a Newtonian formula.

Finally the receiving radar detects a pulse of frequency f_R coming from the direction $\underline{n}_R$ at the instant when its velocity is $\underline{v}_R = c\underline{\beta}_R$ relative to the geocentric inertial observer. By using Eqs. (2), (4), and then (3) f_R can be expressed as

$$f_R = F_R \gamma_R (1 - \underline{\beta}_R \cdot \underline{N}_R)$$

$$= (F_R/\gamma_R)/(1 + \underline{\beta}_R \cdot \underline{n}_R)$$

$$= F_T \gamma_R (1 - \underline{\beta}_R \cdot \underline{N}_R)(1 - \underline{\beta}_S \cdot \underline{N}_T)/(1 - \underline{\beta}_S \cdot \underline{N}_R) \qquad (5)$$

$$= f_T (\gamma_R/\gamma_T) \frac{(1 - \underline{\beta}_R \cdot \underline{N}_R)(1 - \underline{\beta}_S \cdot \underline{N}_T)}{(1 - \underline{\beta}_S \cdot \underline{N}_R)(1 - \underline{\beta}_T \cdot \underline{N}_T)}$$

Lastly the Doppler shift in frequency, $\Delta f = f_R - f_T$, can be expressed as

$$\Delta f/f_T = \left(\frac{1-\beta_T^2}{1-\beta_R^2}\right)^{1/2} \left(\frac{1-\underline{\beta}_S \cdot \underline{N}_T}{1-\underline{\beta}_T \cdot \underline{N}_T}\right) \left(\frac{1-\underline{\beta}_R \cdot \underline{N}_R}{1-\underline{\beta}_S \cdot \underline{N}_R}\right) - 1 \qquad (6)$$

Note that Δf is the difference between two frequencies measured in two different inertial frames and that $\Delta f/f_T$ in Eq. (6) is a quantity expressed in terms of velocities and directions relative to the geocentric inertial reference frame.

Irwin Shapiro* discussed a similar problem in the interplanetary context. In his case the "artificial satellite" was Venus, the transmitting and receiving "radars" were the Earth at the instants of transmission and reception, and the ultimate reference frame was a heliocentric one instead of a geocentric one. To clearly see the correspondence between Shapiro's result and Eq. (6), let $T \rightarrow 1$, $R \rightarrow 3$, $S \rightarrow 2$, $\underline{N}_T \rightarrow \underline{e}_{12}$, and $\underline{N}_R \rightarrow \underline{e}_{23}$.

Finally, when the receiving radar and the transmitting radar are colocated, $\beta_R = \beta_T$ (but $\underline{\beta}_R \neq \underline{\beta}_T$). Hence Eq. (6) reduces to

$$\Delta f/f_T = \left(\frac{1-\underline{\beta}_S \cdot \underline{N}_T}{1-\underline{\beta}_T \cdot \underline{N}_T}\right) \left(\frac{1-\underline{\beta}_R \cdot \underline{N}_R}{1-\underline{\beta}_S \cdot \underline{N}_R}\right) - 1 \qquad (7)$$

Once again, while this formula has the appearance of one which conforms to Galilean relativity, it was derived (in full generality) within the context of a more correct physical theory of nature. Its final simplicity is due

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* I. I. Shapiro, Bull. Astr. XIV, 201, 1965.

to a degenerate geometry (transmitting radar ≡ receiving radar) and the fortuitous cancellation (by division) of two of the four special relativistic terms implicit in Eq. (7).

IV. APPROXIMATIONS

Since the artificial satellite is nearby (or light travels fast) an approximation that appears tempting, when the transmitting and receiving radars are identical, is to ignore the displacement of the radar during the pulse travel time (radar-satellite-radar). The round trip time is $\approx 0^{s}.3$ even for a near-stationary satellite and the Earth's rotation rate is slow ($\approx 15''$/sec so neglecting the displacement is an error of 2 parts in 10^5). If the radar doesn't move (or the pulse is infinitely fast) then it follows that the directions of transmission and reception are antiparallel. Hence, $\underline{N}_T = -\underline{N}_R = N$ and $\underline{n}_T = -\underline{n}_R = \underline{n}$. Furthermore, if the radar's displacement is being neglected, then the approximation that $\underline{\beta}_T = \underline{\beta}_R = \underline{\beta}$ (say at the instant of reflection by the satellite) can also be justified. Now

$$\Delta f/f_T = \left(\frac{1 - \underline{\beta}_S \cdot \underline{N}}{1 - \underline{\beta} \cdot \underline{N}}\right)\left(\frac{1 + \underline{\beta} \cdot \underline{N}}{1 + \underline{\beta}_S \cdot \underline{N}}\right) - 1 \tag{8}$$

Note that ignoring the displacement of the radar during the time of flight of the pulse is not the same as ignoring the relative motion of the radar with respect to the geocentric inertial observer. Were I to do that then I could've set $\underline{\beta} = \underline{0}$ in Eq. (8). This is incorrect and would lead to the neglect of an aberration effect [see Eq. (1)]. This aberration effect is distinct from any parallactic displacement and solely results from the relative motion of the two observers. Moreover, as a glance at Eq. (1) will show, it is a first order effect (e.g., it depends on β).

Since β and β_S are small ($\lesssim 10^{-5}$) a power series expansion of Eq. (8) would seem to be appropriate. The two leading terms are

$$\Delta f/f_T \simeq -2\ \Delta\underline{\beta}\cdot\underline{N} + 2\ (\Delta\underline{\beta}\cdot\underline{N})^2 \qquad (9)$$

where $\Delta\underline{\beta} = \underline{\beta}_S - \underline{\beta}$. $\Delta\underline{\beta}$ is the relative velocity of the artificial satellite with respect to the radar as measured by the geocentric inertial observer. $\Delta\underline{\beta}\cdot\underline{N}$ is the radial velocity of the satellite, relative to the radar, as perceived by the geocentric inertial observer. This is because $\Delta\underline{\beta}$ is not the relative satellite-radar velocity in the radar's comoving reference frame nor is $\underline{N}$ the direction to (the "line-of-sight" of) the satellite in the radar's comoving reference frame ($\underline{n}$ is). The correct radial velocity of the satellite relative to the radar, in the radar's comoving reference frame, can be obtained by performing a Lorentz transformation of $\Delta\underline{\beta}$ and then forming the scalar product of the transformed vector with $\underline{n}$. $\Delta\underline{\beta}$ is the Galilean result for the relative velocity of the satellite with respect to the radar. Finally note that while the difference between $\Delta\underline{\beta}$ and the relative velocity of the artificial satellite with respect to the radar (in the radar's comoving reference frame) is of second order, the difference between $\underline{N}$ and $\underline{n}$ is first order. As the point of this derivation is the full set of second order terms neither of these effects can be neglected. [If you object that $\underline{N}$ only appears in Eq. (9) dotted into $\Delta\underline{\beta}$, and that $\Delta\beta$ is of first order, the net effect of $\underline{N} \neq \underline{n}$ is of the second order in $\Delta f/f_T$. The aberration effect is still of the first order.]

Equation (9) is not in a form suitable for computation because $\underline{N}$ is not observed. The relative velocity $\Delta\underline{\beta} = \underline{\beta}_S - \underline{\beta}$ is expressed geocentrically and this is the coordinate system wherein it is simplest to compute both $\underline{\beta}$ (the radar's geocentric velocity) and $\underline{\beta}_S$ (the artificial satellite's geocentric

velocity). If I return to Eq. (6) and use Eq. (2), then two of the geocentric directions can be eliminated yielding

$$\Delta f/f_T = (\gamma_T/\gamma_R)\left(\frac{1 - \underline{\beta}_S\cdot\underline{N}_T}{1 - \underline{\beta}_S\cdot\underline{N}_R}\right)\left(\frac{1 + \underline{\beta}_T\cdot\underline{n}_T}{1 + \underline{\beta}_R\cdot\underline{n}_R}\right) - 1 \tag{10}$$

The only way I can see to eliminate the other two geocentric inertial unit vectors is to replace them with unit vectors in the artificial satellite's comoving reference frame. As this doesn't facilitate matters, I have demurred.

It is still true that $\beta_T = \beta_R$ if the receiving radar is colocated with the transmitting radar, whence $\gamma_T = \gamma_R$. If in addition I ignore the displacement of the radar during the pulse's transit time, then $\underline{N}_T = -\underline{N}_R = \underline{N}$ and $\underline{n}_T = -\underline{n}_R = \underline{n}$ again. $\Delta f/f_T$ reduces to

$$\Delta f/f_T = \left(\frac{1 - \underline{\beta}_S\cdot\underline{N}}{1 + \underline{\beta}_S\cdot\underline{N}}\right)\left(\frac{1 + \underline{\beta}\cdot\underline{n}}{1 - \underline{\beta}\cdot\underline{n}}\right) - 1 \tag{11}$$

$$\simeq -2\,\underline{\beta}_S\cdot\underline{N} + 2\,\underline{\beta}\cdot\underline{n} - 4\,(\underline{\beta}_S\cdot\underline{N})(\underline{\beta}\cdot\underline{n}) + 2\,(\underline{\beta}\cdot\underline{n})^2 + 2\,(\underline{\beta}_S\cdot\underline{N})^2$$

Taking both β and β_S to be comparably small (in practice $\beta_S \geq \beta$ perhaps $\beta_S \simeq 10\beta$) I can replace $\underline{N}$ by $\underline{n}$ in any quadratic term. After making this approximation I'm left with Eq. (12),

$$\Delta f/f_T \simeq -\,2\,\underline{\beta}_S\cdot\underline{N} + 2\,\underline{\beta}\cdot\underline{n} + 2\,(\Delta\underline{\beta}\cdot\underline{n})^2 \tag{12}$$

Finally, I'm going to write $\underline{N}$ as $\underline{n} + \delta\underline{n}$ where the norm of $\delta\underline{n}$ is of order β. Then the expression for $\Delta f/f_T$ reduces to ($\Delta\underline{\beta} = \underline{\beta}_S - \underline{\beta}$ still)

$$\Delta f/f_T \simeq - 2\, \Delta\underline{\beta}\cdot\underline{n} + 2\, (\Delta\underline{\beta}\cdot\underline{n})^2 - 2\, \underline{\beta}_S\cdot\, \delta\underline{n} \tag{13}$$

The difference between the expression in Eq. (13) and in Eq. (9) clearly shows the effect of aberration.

$\Delta\underline{\beta}\cdot\underline{n}$ is still not the radial velocity of the artificial satellite with respect to the radar (in any reference frame) because $\Delta\underline{\beta}$ is still the relative velocity of the satellite with respect to the radar in the geocentric inertial frame. The fact that $\underline{n}$ is the direction to the artificial satellite in the radar's comoving inertial frame makes $\Delta\underline{\beta}\cdot\underline{n}$ a better approximation to the relative radial velocity than is $\Delta\underline{\beta}\cdot\underline{N}$. To complete the computation a Lorentz transformation of $\Delta\underline{\beta}$ would be required.

If Eq. (13) or Eq. (9) is viewed as the ultimate result of the analysis then the reader should be explicitly aware of two implicit assumptions. One concerns the special theory of relativity, the other the general theory. I have spoken above of a geocentric inertial observer. The implication of such a phrase is that relative to any inertial reference frame the velocity of the geocenter has been constant throughout the pulse travel time (radar-satellite-radar). As the Earth revolves about the Earth-Moon barycenter, and the Earth-Moon barycenter revolves about the solar system's barycenter, and the Sun is accelerated relative to the local standard of rest, and the local standard rest revolves about the galactic center, . . . this hypothesis is known to be false. The numerical consequences of such an assumption will be treated in the next Section.

The general relativistic effect not obviously included concerns the change, if any, between the gravitational potentials of the transmitting and receiving radars. As the practical case has these colocated, the essence of the approximation is to ignore the motions of the other bodies in the solar system during the round trip flight time of the pulse. This too needs to be numerically investigated.

V. NUMERICAL CONSIDERATIONS

There are actually two general relativistic effects that can yield systematic sources of error in the Newtonian reduction of radar observations. The first is the general relativistic Doppler shift mentioned above. The amplitude of the change is proportional to the difference between the gravitational potentials of the transmitting and receiving radars (in the lowest order post-Newtonian approximation). In particular the shift in wavelength $\Delta\lambda$ corresponding to the frequency shift Δf is given by

$$\Delta\lambda/\lambda = \Delta\Phi/c^2$$

where Φ is the negative of the usual Newtonian gravitational potential. For a particle on the Earth's surface $\Phi \approx GM_E/R_E$ but this expression ignores both the oblateness of the Earth and third-body perturbations. Consider third-body perturbations due to an object of mass M at a geocentric distance D. Since the (ground based) radars can at most be separated by $2R_E$,

$$c^2\Delta\lambda/\lambda \leq \frac{GM}{D-R_E} - \frac{GM}{D+R_E} \approx 2GMR_E/D^2$$

When the third-body is the Moon $\Delta\lambda/\lambda \leq 4.7 \times 10^{-15}$, when it's the Sun $\Delta\lambda/\lambda \leq 8.4 \times 10^{-13}$.

The change in the gravitational potential due to the effects of the Earth's oblateness can be much larger. Using the expression

$$\frac{-GM_E}{r}\left[1 - \frac{J_2P_2(\sin\phi)}{r^2}\right]$$

for the geopotential (J_2 = 0.00108, ϕ = terrestrial latitude), it follows that

$$c^2\Delta\lambda/\lambda \lesssim 2GM_E\, J_2/R_E^{\,3}$$

Hence, $\Delta\lambda/\lambda \lesssim 3.3 \times 10^{-9}$ which is significantly larger than the second order special relativisitic effect due to the Earth's rotation ($\omega_E = 15''.041$/sec, R_E = 6378.1 km so β_E = 0.47 km/sec/c = 1.6×10^{-6}, $\beta^2 = 2.4 \times 10^{-12}$).

The second general relativistic effect is due to the lack of flatness (in Minkowski space) of the general relativistic line element near massive bodies. This results in a departure of the speed of light near such an object from its speed in vacuo (eg. c). Hence distance to an object, as deduced by halving the round trip flight time and then dividing by c, must be corrected. In the lowest order post-Newtonian approximation the amount of the correction is equal to the path integral of the (Newtonian) gravitational potential. Thus,

$$\Delta_s = (2/c)\int \Phi dt \approx (2/c)\int \frac{GM_E}{r}\,\frac{dr}{c}$$

$$\approx (2GM_E/c^2)\,\ell n\,(R_{sat}/R_E)$$

This amounts to $\sim$ 1.7mm for a near-stationary satellite.

There are still two approximations within the context of a special relativistic computation that await numerical investigation. One is $\underline{\beta}_R = \underline{\beta}_T$ and the other is $\underline{N}_R = -\underline{N}_T$, $\underline{n}_R = -\underline{n}_T$. Consider the velocity approximation first when the transmitting and receiving radars are colocated. Then the difference between $\underline{\beta}_R$ and $\underline{\beta}_T$ is the result of a change of direction due to the rotation of the Earth during the pulse's travel time. Clearly this is of order $\Delta\beta = \omega_E\Delta t$ with $\Delta t \approx 0^s.1$ whence we are speaking of an effect of order

$10^{-5}\ \beta_E$ ($0\overset{s}{.}1$ in radians is $\sim 10^{-5}$). In turn the lowest order (in β_E) that this correction can occur is first. Hence the $\underline{\beta}_R = \underline{\beta}_T$ approximation is of order $\beta_E \Delta\beta \sim 10^{-5}\beta_E^2$. Therefore, in any analysis good to $\beta^2{}_E$, this term can be neglected. An analytical refinement of this argument, directly from Eq.(7) or Eq. (11), leads to the same result.

The other approximation is the neglect of an aberration difference. $\underline{n}_R$ is not anti-parallel to $\underline{n}_T$ because they're measured in two different inertial frames (not because of the parallactic displacement of the radars). But the size of this difference, cf Eq. (1) or (2), is of the order of the velocity difference between the two reference frames. This difference, when the transmitting radar is colocated with the receiving radar, has just been shown to be of order $10^{-5}\beta_E$ in magnitude. As the unit vectors $\underline{n}_R$ and $\underline{n}_T$ only appear in scalar products with quantities (at most) of order (10) β_E, it therefore follows that the neglect of this aberration term is of order $10^{-5}(10^{-4})\beta^2{}_E$. Thus, it too is negligible in a $\beta^2{}_E$ analysis.

A final caution. While I have not hesitated to evaluate various post-Newtonian correction terms in a Newtonian manner, this is very different from looking at Eq. (7) and deciding that an ab initio Newtonian derivation of it is appropriate. That would be wrong and can clearly never lead to a post-Newtonian term to evaluate. Much muddled thinking can be avoided if the problem is rigorously solved first and then familiar concepts used to describe the result.

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This Report discusses the special relativistic and general relativistic effects on electromagnetic propogation for artificial satellite applications. In particular the rigorous special relativistic Doppler shift between the received and transmitted frequencies is derived. The two radars need not be colocated and are assumed not to be in the same inertial reference frame. Numerical estimates of various approximations, second order series approximations, and lowest order post-Newtonian general relativistic terms are all performed. This work was spurred by the observational determination of the $(v/c)^2$ term in Millstone Hill Radar observations of LAGEOS.

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